# Signal Fluctuations and Noise for Imaging CoHerent Optical Radar (ICHOR)

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# Outline

- Objective
- Coherent Detection
  - Non-Fluctuating Signal + Noise
  - Fluctuating Signal + Noise
    - Speckle
    - Speckle with Multiple Measurement Integration
    - Speckle and Turbulence
  - Comparison
- Photon Counting (Geiger-mode APD) Detectors
  - Non-Fluctuating Signal + Noise
  - Fluctuating Signal + Noise
    - Speckle
    - Speckle and Turbulence
  - Comparison
- Discussion

# Objective

- Discuss the Signal + Noise Models appropriate for use in performance modeling of coherent detection laser receiversand of photon counting (Geiger-mode) laser receivers

- Indicate the amount of increased signal required to overcome speckle and turbulence induced signal fluctuations by comparing the Probability of Detection vs. SNR curves

- Discuss using Multiple Independent Measurements to Overcome Signal Fluctuations

# Reference

- The analysis uses results from Osche, Gregory R., Optical Detection Theory for Laser Applications, Hoboken, New Jersey: John Wiley & Sons, 2002.
  - Treats Coherent Detection
  - Direct Detection
    - Linear Response Detectors
    - Photon Counting Detectors including Geiger-mode APDs
  - Speckle
  - Weak Turbulence including the backscatter amplification for monostatic receivers
  - Multi-pulse Averaging

# Coherent Receiver Probability of Detection Equations

Coherent Detection with Linear Response Detectors:

Non-fluctuating Signal + AWGN single pulse:

$$POD_{NFS\_AWGN_i} := 1 - \int_0^{TNR} \exp\left[-\left(r + SNR_i\right)\right] \cdot I0\left(2\sqrt{r \cdot SNR_i}\right) dr$$

Speckle Signal + AWGN:

$$POD_{Speckle\_AWGN_i} := P_{fa}^{\frac{1}{1+SNR_i}} \qquad POD_{SwerlingII_i} := \frac{\Gamma\left[N_p, \frac{TNR}{2\cdot\left(1 + SNR_i\right)}\right]}{\Gamma\left(N_p\right)} \qquad P_{fa\_mp} := \frac{\Gamma\left(N_p, \frac{TNR}{2}\right)}{\Gamma\left(N_p\right)}$$

Speckle + Turbulence Signal + AWGN single pulse, bistatic receiver:

$$POD_{Speckle\_Turbulence\_AWGN\_bistatic_i} := \frac{1}{\sqrt{2\cdot\pi\cdot\sigma_\chi^2}} \cdot \int_{-300}^{300} \exp\left[-\frac{\left(\chi + \sigma_\chi^2\right)^2}{2\cdot\sigma_\chi^2}\right] \cdot P_{fa}^{\frac{1}{1+SNR_i\cdot\exp(2\cdot\chi)}} d\chi$$

Speckle + Turbulence Signal + AWGN single pulse, monostatic receiver:

$$POD_{Speckle\_Turbulence\_AWGN\_monostatic_i} := \frac{1}{\sqrt{2\cdot\pi\cdot\sigma_\chi^2}} \cdot \int_{-100}^{100} \exp\left[-\frac{\left(\chi + \sigma_\chi^2\right)^2}{2\cdot\sigma_\chi^2}\right] \cdot P_{fa}^{\frac{1}{1+SNR_i\cdot\exp(4\cdot\chi)}} d\chi$$

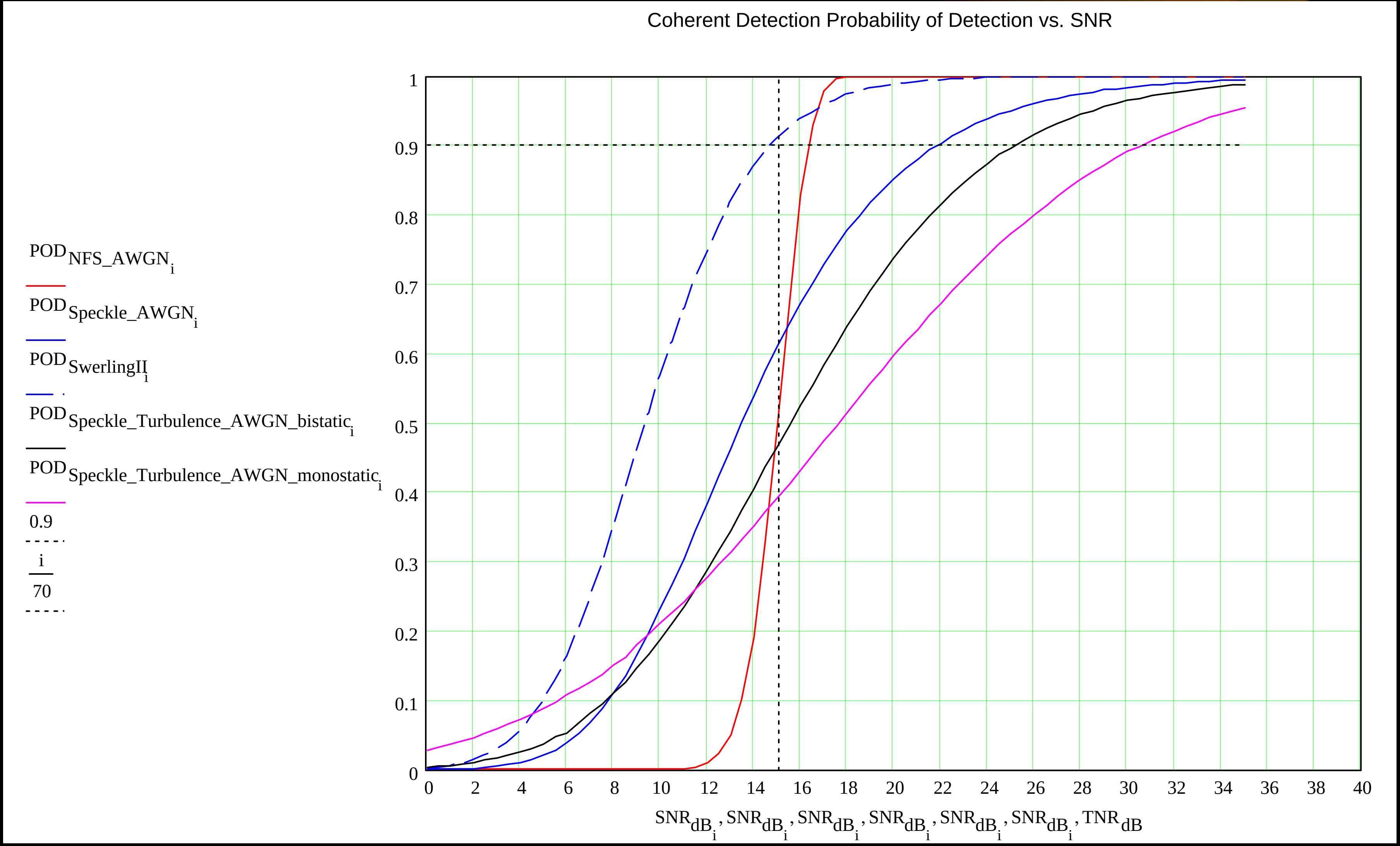
# Coherent Receiver Probability of Detection Parameter Values for Sample Calculations

$\lambda := 1.5 \cdot 10^{-6}$ $k := \frac{2\cdot\pi}{\lambda}$ $R := 1000$ $c := 3\cdot 10^{8}$ $P_{fa} := 10^{-7}$ $Cn2 := 10^{-13}$ $i := 0..70$ $P_{fa\_mp} = 1.712\times 10^{-6}$

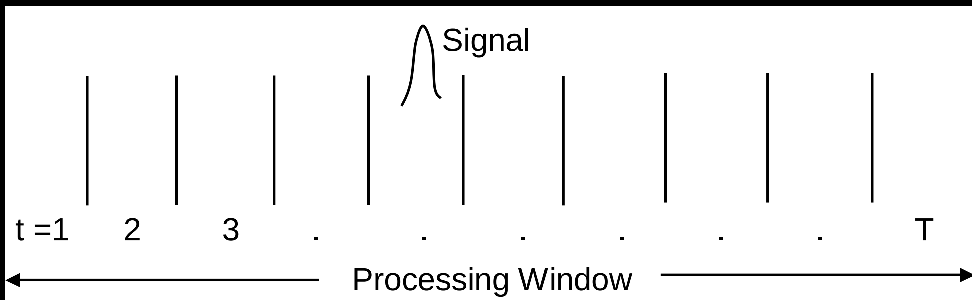
$\sigma_{\chi\_pw} := \sqrt{\frac{1.23}{4}\cdot Cn2\cdot k^{\frac{7}{6}}\cdot R^{\frac{11}{6}}}$ $\sigma_{\chi\_sw} := \sqrt{\frac{0.496}{4}\cdot Cn2\cdot k^{\frac{7}{6}}\cdot R^{\frac{11}{6}}}$ $\sigma_{\chi\_pw} = 0.719$ $\sigma_{\chi\_sw} = 0.457$ $\sigma_{\chi} := \sigma_{\chi\_sw}$ $\sigma_{\chi} = 0.457$

$SNR_i := 10^{\frac{i}{20}}$ $TNR := -2\cdot\ln(P_{fa})$ $TNR = 32.236$ $N_p := 2$ $SNR_{dB_i} := 10\cdot\log(SNR_i)$ $TNR_{dB} := 10\cdot\log(TNR)$ $TNR_{dB} = 15.083$

# Coherent Receiver Probability of Detection Results



# Photon Counting Probability of Detection Equations




**Multiple sequential measurement intervals over a time period called a Processing Window. The Processing Window is divided into T equal measurement intervals, in one of which may reside the signal**

Photon Counting:

Non-fluctuating Signal and Photon Counting:

$$POD_{NFS\_PC_i} := 1 - e^{-\left(N_{s_i} + N_n\right)}$$

$$P_{fa\_PC} := 1 - e^{-N_n} \qquad P_{fa\_PC} = 9.95 \times 10^{-3}$$

$$POD_{NFS\_PC\_T_i} := POD_{NFS\_PC_i} \cdot \left(1 - P_{fa\_PC}\right)^{T-1}$$

$$P_{fa\_NFS\_PC\_1_i} := \left(1 - POD_{NFS\_PC_i}\right) \cdot \left[1 - \left(1 - P_{fa\_PC}\right)^{T-1}\right]$$

$$P_{fa\_PC\_T} := \left[1 - \left(1 - P_{fa\_PC}\right)^{T-1}\right]$$

Speckle Signal & Photon Counting:

$$POD_{Speckle\_PC_i} := 1 - \left(\frac{M}{M + N_{s_i}}\right)^{M} \cdot e^{-N_n}$$

$$POD_{Speckle\_PC\_T_i} := POD_{Speckle\_PC_i} \cdot \left(1 - P_{fa\_PC}\right)^{T-1}$$

$$P_{fa\_Speckle\_PC\_1_i} := \left(1 - POD_{Speckle\_PC_i}\right) \cdot \left[1 - \left(1 - P_{fa\_PC}\right)^{T-1}\right]$$

# Photon Counting Probability of Detection Equations

Speckle + Turbulence Signal + Photon Counting, bistatic receiver:

$\sigma_l := 2\cdot\sigma_\chi$  $\sigma_l^2 = 0.834$

$$POD_{Speckle\_Turbulence\_PC\_bs_i} := 1 - \frac{1}{\sqrt{2\cdot\pi\cdot\sigma_l^2}}\cdot\int_{-250}^{250}\frac{1}{\left(1 + N_{s_i}\cdot e^{l} + N_n\right)}\cdot\exp\left[\frac{-\left(l + \frac{\sigma_l^2}{2}\right)^2}{2\cdot\sigma_l^2}\right]dl$$

$$POD_{Speckle\_Turbulence\_PC\_T\_bs_i} := POD_{Speckle\_Turbulence\_PC\_bs_i}\cdot\left(1 - P_{fa\_PC}\right)^{T-1}$$

$$P_{fa\_Speckle\_Turbulence\_PC\_1\_bs_i} := \left(1 - POD_{Speckle\_Turbulence\_PC\_bs_i}\right)\cdot\left[1 - \left(1 - P_{fa\_PC}\right)^{T-1}\right]$$

Speckle + Turbulence Signal + Photon Counting, monostatic receiver:

$$POD_{Speckle\_Turbulence\_PC\_ms_i} := 1 - \frac{1}{\sqrt{2\cdot\pi\cdot\sigma_l^2}}\cdot\int_{-250}^{250}\frac{1}{\left(1 + N_{s_i}\cdot e^{2l} + N_n\right)}\cdot\exp\left[\frac{-\left(l + \frac{\sigma_l^2}{2}\right)^2}{2\cdot\sigma_l^2}\right]dl$$

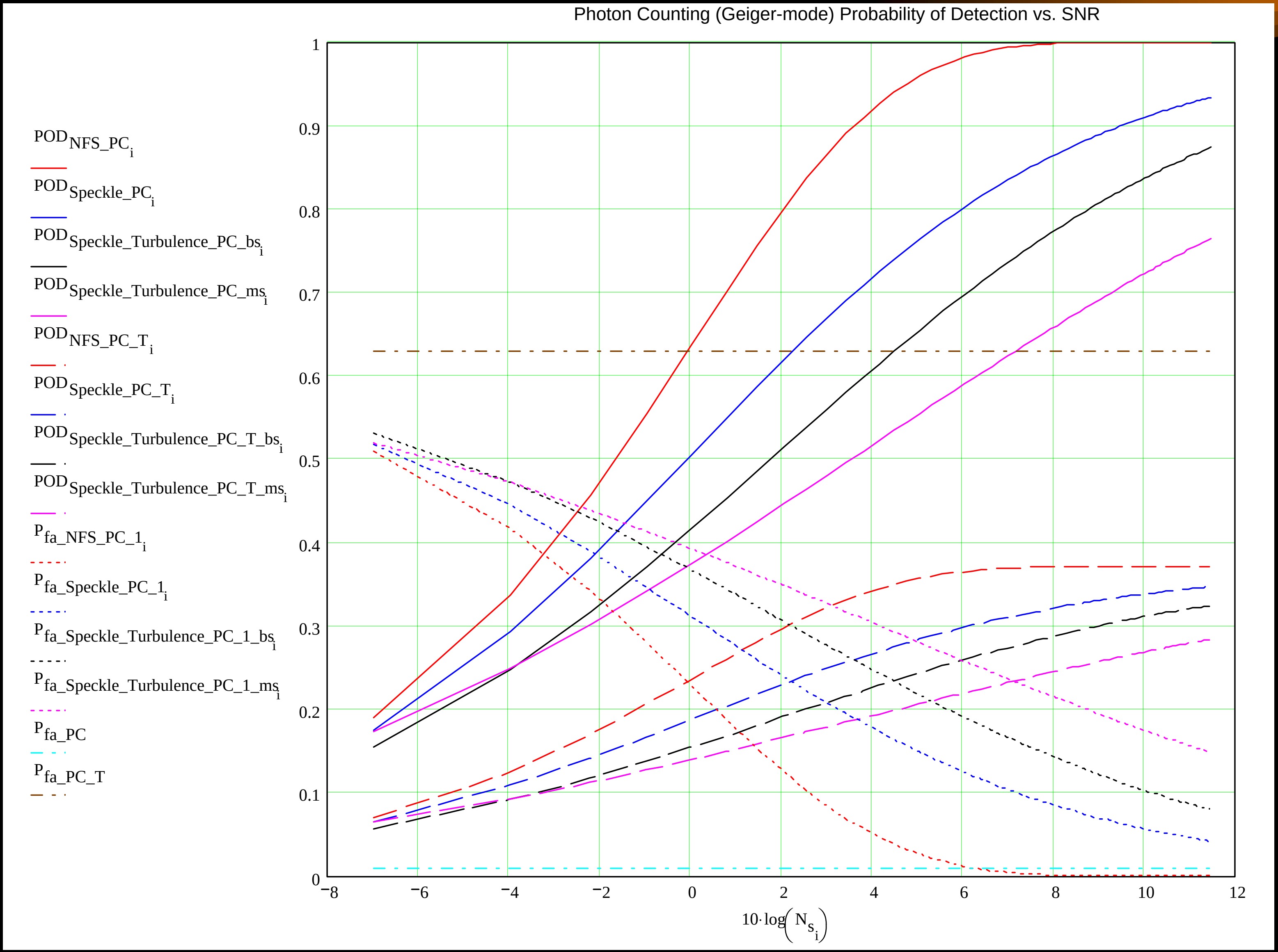
$$POD_{Speckle\_Turbulence\_PC\_T\_ms_i} := POD_{Speckle\_Turbulence\_PC\_ms_i}\cdot\left(1 - P_{fa\_PC}\right)^{T-1}$$

$$P_{fa\_Speckle\_Turbulence\_PC\_1\_ms_i} := \left(1 - POD_{Speckle\_Turbulence\_PC\_ms_i}\right)\cdot\left[1 - \left(1 - P_{fa\_PC}\right)^{T-1}\right]$$

# Photon Counting Probability of Detection
## Parameter Values for Sample Calculations

$M := 1$ $N_{s_i} := \frac{i}{5}$ $N_n := 0.01$ $k_{th} := 1$ $T := 100$ $N_n \cdot T = 1$

# Photon Counting Probability of Detection Results



# Discussion

- Coherent Receivers
  - Overcoming Speckle and Turbulence can require large increases in laser power (>3dB) for a single measurement, OR
  - Integration over Multiple Measurements
    - If the time between measurements is greater than the correlation time of the statistical process (e.g., Swerling Case II), the POD for N measurements can be greater than for a single measurement with N times the energy of each of the multiple measurements
    - Osche shows that for the Swerling Case II with Pfa = 1E-08 and POD = 0.9 (Figure 6-24), integration over multiple measurements can require ~ 5 dB less energy per measurement than the single measurement energy divided by N, for N ~ 5 - 10 measurements, i.e., there is a ~ 5 dB improvement compared to coherent integration
  - In Turbulence, the backscatter amplification effect degrades the POD for the monostatic receiver compared to the bistatic receiver
- Photon Counting Receivers
  - Overcoming Speckle and Turbulence can require large increases in laser power (>3dB), OR
  - Multiple Measurements can be used to overcome the loss, but since the averaging is post-detection and the signal is binary, m out of n processing (aka, coincidence detection or double-thresholding) is used