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Modelling and Simulation of Acoustic Wave Propagation in Locally Resonant Sonic Materials

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Abstract

Sonic crystals are artificial structures consisting of a periodic array of acoustic scatterers embedded in a homogeneous matrix material, with a usually large impedance mismatch between the two materials. They exhibit strong sound attenuation at selective frequency bands due to the interference of multiply reflected waves. However, sound attenuation bands in the audible range are only achieved by unfunctionally large sonic crystals. If local resonators are used instead of simple scatterers, the frequencies of the attenuation bands can be reduced by about two orders of magnitude.

In the present paper we perform numerical simulations of acoustic wave propagation through sonic crystals consisting of local resonators using the Local Interaction Simulation Approach (LISA). Three strong attenuation bands are found at frequencies between 0.3 and 6.0 kHz, which do not depend on the periodicity of the crystal. The results are in good qualitative agreement with experimental data. We analyze the dependence of the resonance frequencies on the structural parameters of the local resonators in order to create a tool for design and optimization of any kind of sonic crystal.

Keywords: Sonic crystals, Numerical simulations, Sound attenuation, Local resonances, Audible frequency range, Modelling

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1. Introduction

Usually, the goal in the design of sound insulation material is to achieve strong sound attenuation over a large frequency range. However, for application as acoustic filters or noise suppression, materials that impede the propagation of acoustic waves only at selected frequency bands may be desirable.

Similar effects are well known for electrons in natural crystals [1] and for electromagnetic waves in photonic crystals [2, 3]. Martínez-Sala et al. [4] found that a periodic array of acoustic scatterers in a homogeneous matrix material (e. g. solid cylinders in air) exhibits similar effects on acoustic waves, due to superposition of multiply reflected waves within the array, according to Bragg's theory. In the following years a large number of experimental [5-8] and theoretical investigations [9-10] have been carried out, leading to analogous results for diverse types of

periodic structures, thus called *Sonic Crystals*. These studies have shown that the existence of a pronounced sound attenuation band is strongly connected with a large acoustic impedance ratio between the scatterers' and the matrix' material. The band width and depth vary with the density of the scatterers inside the sonic crystal. The center frequencies, however, are always given by Bragg's condition

$$f = n \cdot \frac{v}{2a} \qquad (n = 1, 2, \ldots), \tag{1}$$

where v is the longitudinal sound velocity of the matrix material and a is the lattice constant (i. e. the center distance between adjacent acoustic scatterers). These results have been confirmed by numerical simulations using the Local Interaction Simulation Approach (LISA) [11, 12], in good qualitative agreement with experimental and theoretical data [13].

In order to obtain sound attenuation bands in the audible frequency range, the dimensions of the sonic crystals become too large to be suitable as sound insulation material. Furthermore, the sound attenuation depends on the direction of the wave propagation relative to the symmetry axes of the array. These shortcomings have been overcome by Liu et al [14], who built a sonic crystal consisting of an array of acoustic scatterers, which exhibit resonance frequencies themselves. These local resonators are silicone rubber coated metal spheres. Silicone rubber is a very weak material, which allows the metal sphere to vibrate within its coating. Thus, the sound attenuating frequency bands are not determined by the scatterers' distribution, but by their intrinsic structure. Experimental measurements of the sound transmission through those locally resonant sonic crystals yield strong attenuation bands at frequencies about two orders of magnitude lower than predicted by Bragg's theory (Eq. 1). The results have been verified numerically using two-dimensional LISA simulations providing good qualitative agreement with the experiment. The effect is found to be due to the individual scatterers [13]. At low frequencies, the depth of the sound attenuation bands (in dB) increases proportionally with the number of consecutive scatterers passed by the incoming acoustic wave. At higher frequencies, sound attenuation due to Bragg reflections superposes the

local resonances. Periodic arrays of local resonators are used in the experiment and in the LISA simulations in order to obtain strong and well defined attenuation bands.

In the present paper we perform numerical LISA simulations to calculate the sound attenuation of a sonic crystal consisting of an array of silicone rubber coated hollow steel cylinders (see Figure 1) embedded in an epoxy matrix. Hollow cylinders have been selected for obvious reasons of economy and weight reduction. The simulation technique and setup of the virtual experiment are briefly described in Section 2. Three different modes of local resonances occur in the analyzed frequency range from 0.3 to 6.0 kHz. We study the frequency dependence of these resonances on the structural parameters of the local resonators and of the sonic crystal. The results are presented in Section 3.

2. Simulation Setup

The LISA approach has been proven to be an efficient tool for the numerical simulation of the acoustic wave propagation in heterogeneous material specimens, in particular those with sharp boundaries between different materials, like in sonic crystals. Its reliability has been demonstrated in a large number of 1-, 2-, and 3-dimensional virtual experiments, yielding excellent agreement with analytical and experimental results [11]. In the present study we restrict ourselves to a 2-D model, since a 3-D simulation would increase drastically the computer time. However, the effects of sonic crystals and locally resonant sonic materials have been well reproduced [13].

Based on the formalism of Finite Difference Equations (FDE), sufficiently small spatial (ε) and temporal (τ) discretization steps must be adopted, e. g. $\varepsilon \leq \lambda/20$ and $\tau \leq \varepsilon/v$ (v = sound propagation velocity), in order to guarantee numerical stability of the algorithm and to reproduce reasonably the “cylindrical” shape of the local resonators. On the other hand, ε cannot be too small since the computer time increases inversely with the third power of ε. Since we calculate the sound transmission in the audible frequency range, λ is very large and the only limiting factor is the

dimension of the local resonator. We set the spatial discretization step to $\varepsilon = 0.3125$ mm, which corresponds to 1/48 of the total diameter of one local resonator (see Table 1). The temporal discretization step is calculated for the material with the highest sound propagation velocity. The LISA simulation setup is shown in Figure 2. We use periodic boundary conditions in the Y direction in order to reduce the computer time and to be able to inject plane waves. Our sonic crystal consists of 8 infinite columns of local resonators. The elastic constants of the used materials are summarized in Table 2.

Plane monochromatic waves propagate in the X direction through the sonic crystal, after having been injected by an input transducer on the left side of the crystal. The displacement averaged over Y is registered at both sides of the crystal. The sound attenuation coefficient is obtained from the ratio of the displacements at the receiver and at the input transducer. In order to avoid unphysical reflections from the borders of the specimen, we have implemented second order absorbing boundary conditions in the X direction [15].

In the case of the usual sonic crystals, whose sound attenuation is based on the superposition of multiply reflected waves, it is essential to calculate a large number of periods of the transmitted wave, in order to obtain the correct transmission coefficient. For sonic crystals consisting of local resonators, we can reduce the total number of time steps in the simulation, since the attenuation is mostly due to local effects within the scatterers. However, it takes some time until the oscillations become stationary. Therefore we calculate for each frequency about 20 periods of the transmitted wave. In spite of the mentioned simplifications, for a large series of frequencies, the simulation becomes very expensive. For the calculations presented in this paper we have used a parallel code specially adapted for multi-processor systems.

3. Results and Discussion

For a better understanding of the experimentally [14] and numerically [13] observed local resonances and in view of a projected design optimization, we vary the most relevant structural parameters of the sonic crystal: lattice constant *a*, inner radius of the steel cylinder r_i, and width of the silicone rubber coating Δr. The variations are performed relatively to the reference parameters shown in Table 1. For each case we calculate the sound attenuation coefficient for a series of frequencies in the range from 0.3 to 6.0 kHz. The elastic constants of the materials are shown in Table 2. Silicone rubber has very small elastic constants and consequently very low sound propagation velocity. Its weakness is the basic ingredient for the existence of local resonances, since it facilitates the vibration of the steel cylinders.

Variation of the lattice constant

The simulation of the sound propagation through the sonic crystal with the reference parameters listed in Table 1 yields three strong attenuation peaks centered around 0.7 kHz, 1.6 kHz, and 4.9 kHz, respectively, the latter being a double peak with a satellite attenuation maximum around 5.2 kHz (see Figure 3). The frequencies are not affected by variations of the lattice constant, which suggests that they are entirely due to local effects within the individual scatterers. We observe a uniform decrease of the sound attenuation with increasing lattice constant, due to the decreasing density of the local resonators. The attenuation at the 5.2 kHz peak decreases much more compared to the other frequencies. Therefore, we expect that is due not only to local effects, but also to interactions between neighboring scatterers. For the reference value of the lattice constant (almost close packing) the lowest resonance frequency is about two orders of magnitude lower than predicted by Bragg's law (Eq. 1).

Variation of the inner radius of the steel cylinder

Vasseur et al [8] have shown both experimentally and theoretically that the sound attenuation of a sonic crystal composed of a periodic array of copper cylinders in air is the same for

hollow and filled cylinders. This happens because the considered wavelengths are much larger than the structure of individual cylinders. A hollow metal cylinder cannot generate any local resonance phenomena either. The elastic constants of steel are very similar to the ones of copper, hence we expect the same independence from the inner radius for our steel cylinders, if we ignore the silicone rubber coating. Certainly the local resonance due to the rubber coating will still be present, but by varying the inner radius of the steel cylinders we effectively vary only their mass.

Figure 4 shows that the lowest resonance frequency decreases with the inner radius r_i of the steel cylinder (i. e. with increasing cylinder mass), whereas the other resonance frequencies do not change at all. Even the absolute sound attenuation remains almost constant for the whole frequency range apart from the lowest attenuation peak. This suggests that at the lowest resonance frequency the steel cylinders vibrate as a whole within the silicone rubber coating, whereas they remain at rest at the other resonance frequencies, with the vibration limited to the coating. This assumption is confirmed by the effect of the variation of the coating's width.

Variation of the width of the silicone rubber coating

In this simulation series not only the width of the coating is changed but also the outer and inner radius of the steel cylinders, so that the thickness of the cylinder shells ($r_o - r_i = 1.25$ mm) remains constant. The two higher resonance frequencies are not affected by the resulting mass change of the steel cylinder, so that they allow us to observe the effect of changing the width of the silicone rubber coating. Figure 5 shows that those resonance frequencies decrease in a similar way with increasing coating width. The lowest resonance frequency, however, changes much less due to the competing effects of increasing coating width and decreasing cylinder mass.

4. Conclusions

Sonic crystals consisting of local resonators yield strong attenuation bands at selected frequencies, with a far superior performance with respect to the usual sonic crystals (based on

Bragg's scattering only). In fact, they succeed in decreasing the peak frequency of up to a factor 100, with similar relative sound attenuation as for usual sonic crystals. Their properties were first discovered experimentally by Liu et al. [14], using silicone rubber coated lead spheres as local resonators.

In order to cut down computer expenses, we have restricted ourselves to 2-D simulations, thus implicitly considering coated hollow cylinders, instead of spheres. Yet our results, obtained by means of the LISA approach, are in good qualitative agreement with the findings of Liu et al. [14]. In fact, by using (virtual) specimens of just about 12 cm of size, we obtain in the audible range three well marked attenuation peaks (between 0.3 and 6.0 kHz) with a relative attenuation between 15 and 25 dB. We have analyzed the dependence of the three peaks on the relevant geometrical parameters of the resonators and the crystal. Our results may help to gain a better insight of the mechanisms governing local resonances. From an applicative point of view, they can be used to predict the structural parameters needed to fabricate custom-tailored sonic crystals.

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Tables

	ε	r_i	r_o	Δr	a
[mm]	0.3125	3.75	5.0	2.5	15.9375
[ε]	1	12	16	8	51

Table 1. *Structural parameters of the sonic crystal in mm and units of ε: ε, r_i, r_o, Δr, and a are the spatial discretization step, internal and external radii of the cylindrical steel shell, width of the silicone rubber coating, and lattice constant, respectively.*

Material	Epoxy	Steel	Silicone Rubber	Air
λ [GPa]	4.43	119.4	$6\cdot10^{-4}$	$1.42\cdot10^{-4}$
μ [GPa]	1.59	79.6	$4\cdot10^{-5}$	0
ρ [kg/m^3]	1180	7780	1300	1.23
v_L [m/s]	2540	5985	22.9	340
v_T [m/s]	1161	3199	5.5	0

Table 2. *Lamé constants λ and μ, density ρ, longitudinal v_L and transverse v_T sound velocities of the considered materials. The very low values of λ and μ (and consequently of v_L and v_T) refer to very soft rubber, as used in the experiments of ref. 14 [16].*

Figure Captions

Figure 1. Local resonator composed of a hollow steel cylinder (outer radius r_o, inner radius r_i) coated by a layer of silicone rubber (width Δr).

Figure 2. LISA simulation setup for a sonic crystal consisting of 8 x ∞ local resonators.

Figure 3. Sound transmission through a sonic crystal consisting of 8 x ∞ local resonators for various values of the lattice constant. Full circles here and in the following figures correspond to the reference case.

Figure 4. Sound transmission through a sonic crystal consisting of 8 x ∞ local resonators for various values of the inner steel cylinder radius.

Figure 5. Sound transmission through a sonic crystal consisting of 8 x ∞ local resonators for various values of the width of the silicone rubber coating.

Figure 1

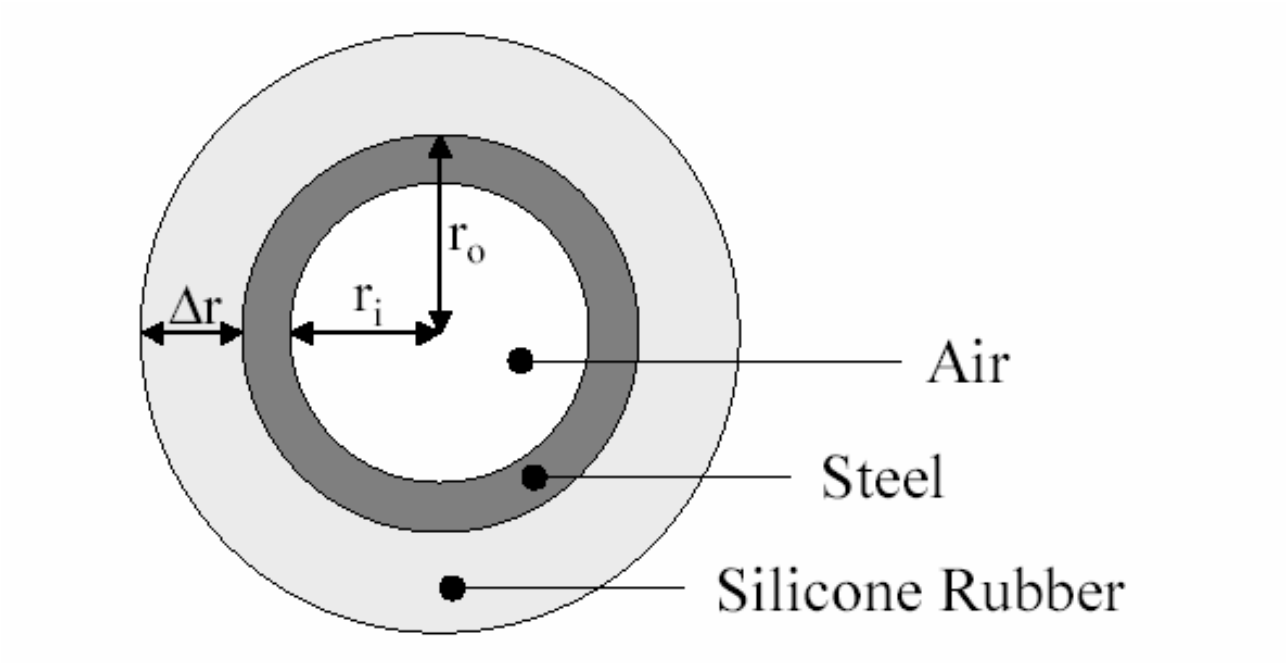


Figure 2

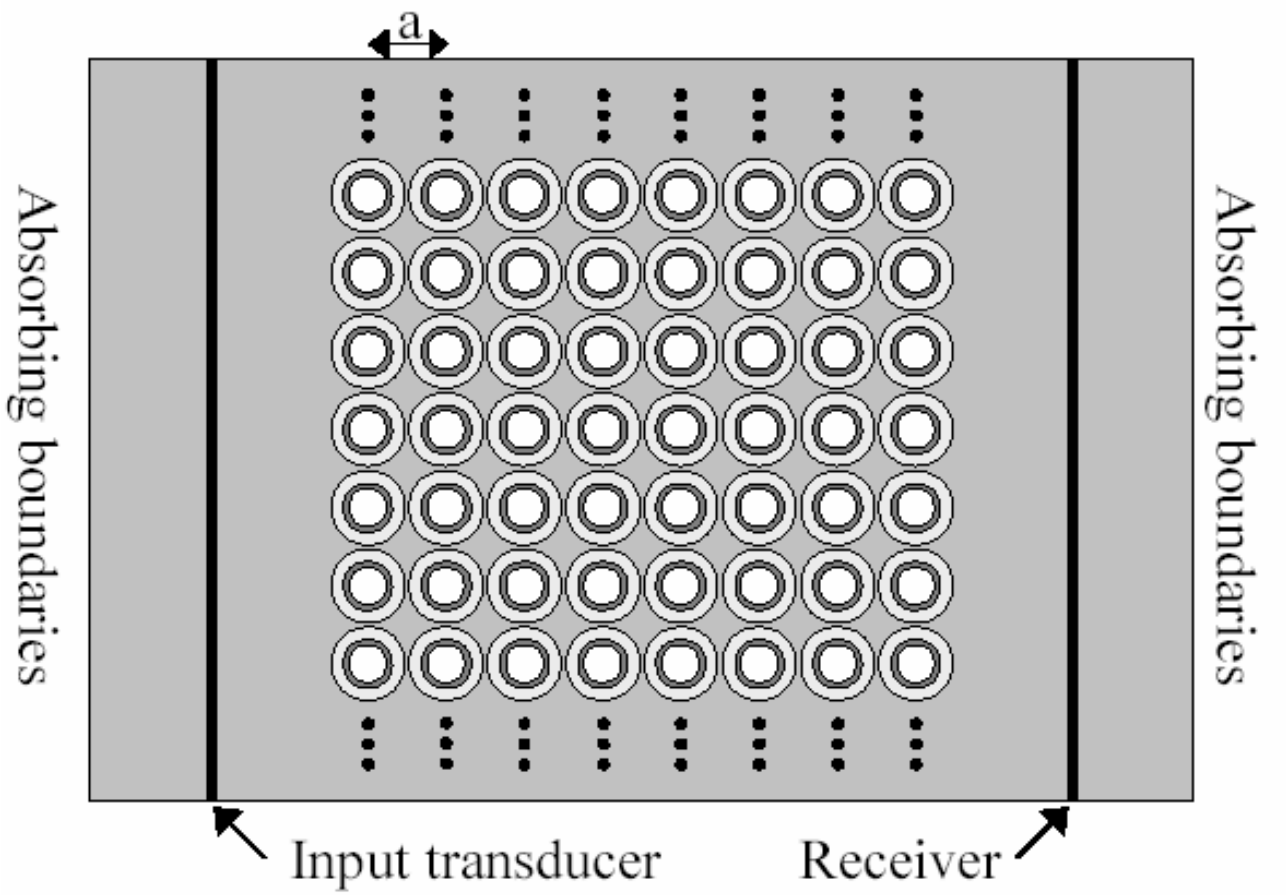

Absorbing boundaries
Input transducer
a
Receiver
Absorbing boundaries

Figure 3

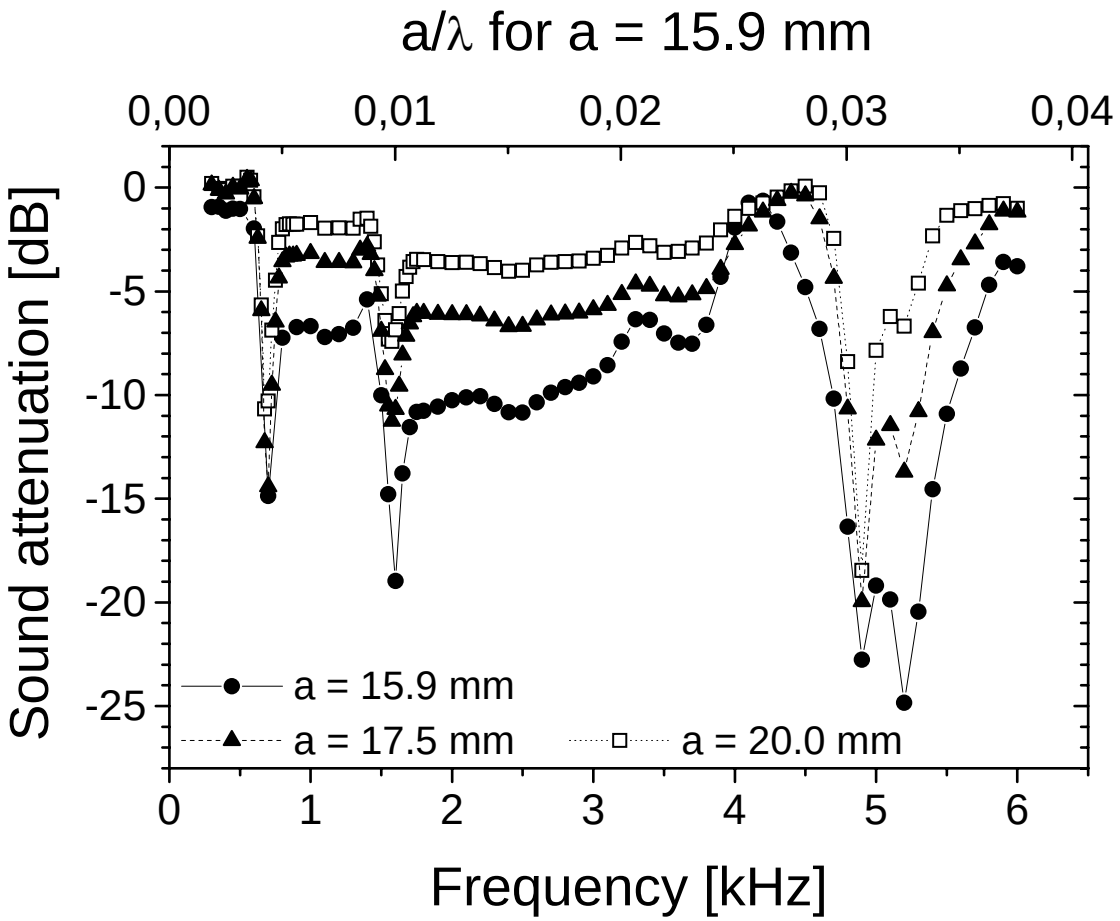

a/λ for a = 15.9 mm
0,00
0,01
0,02
0,03
0,04
Sound attenuation [dB]
0
-5
-10
-15
-20
-25
a = 15.9 mm
a = 17.5 mm
a = 20.0 mm
0
1
2
3
4
5
6
Frequency [kHz]

Figure 4

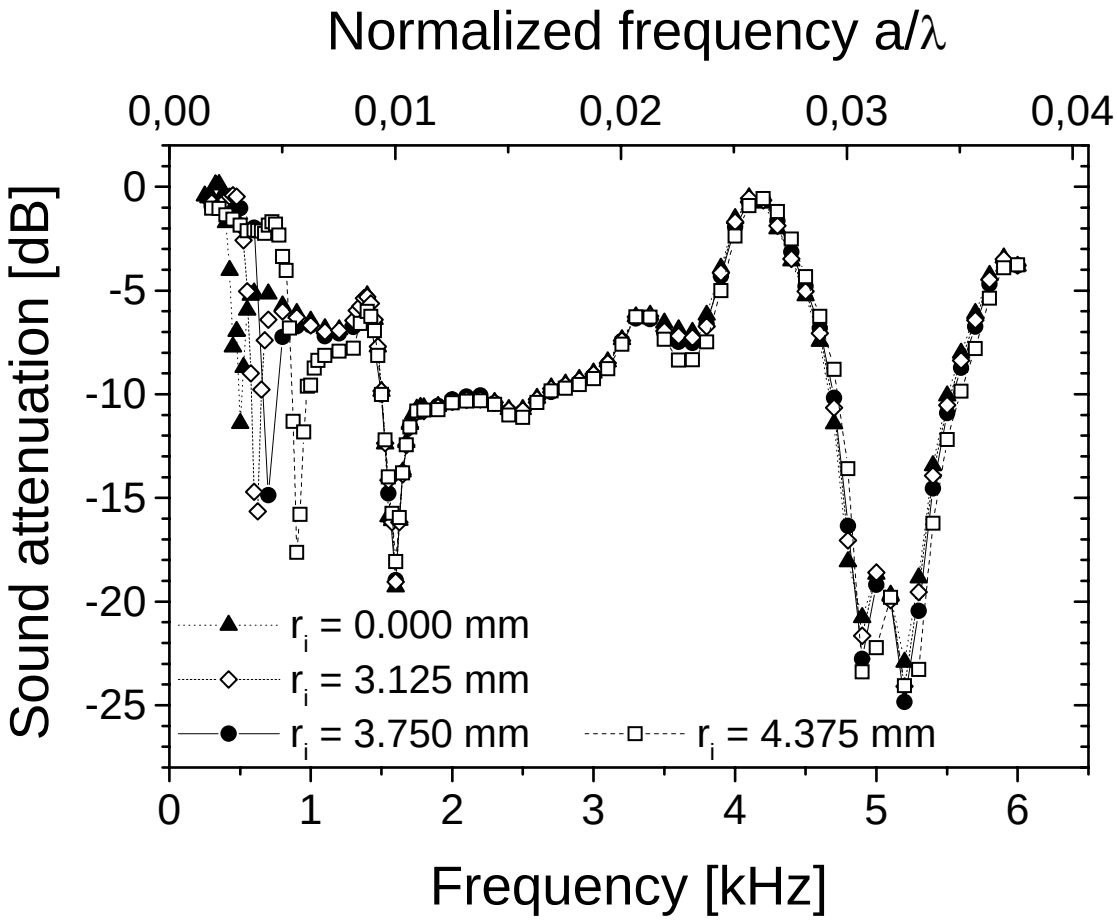
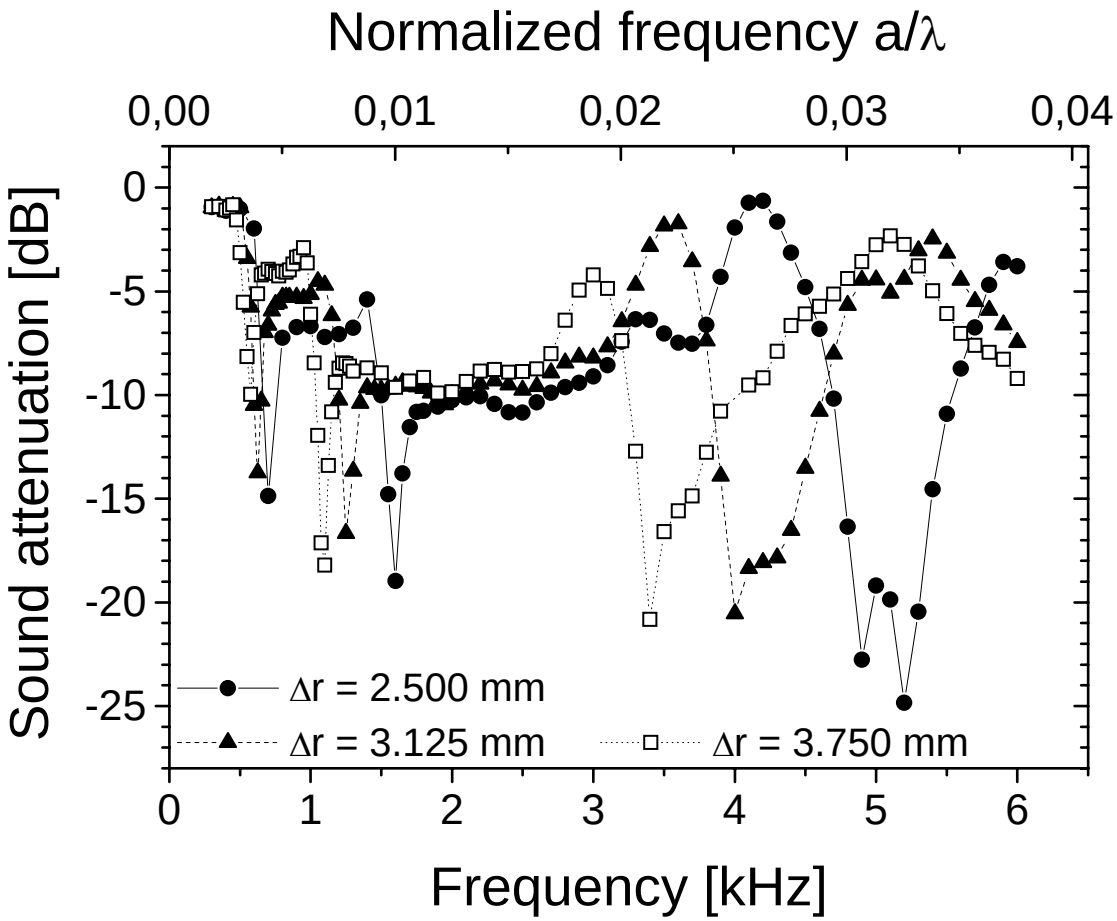

Normalized frequency a/λ
0,00
0,01
0,02
0,03
0,04
Sound attenuation [dB]
0
-5
-10
-15
-20
-25
r_i = 0.000 mm
r_i = 3.125 mm
r_i = 3.750 mm
r_i = 4.375 mm
0
1
2
3
4
5
6
Frequency [kHz]

Figure 5


Normalized frequency a/λ
0,00
0,01
0,02
0,03
0,04
Sound attenuation [dB]
0
-5
-10
-15
-20
-25
Δr = 2.500 mm
Δr = 3.125 mm
Δr = 3.750 mm
0
1
2
3
4
5
6
Frequency [kHz]